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# NATURAL CONVECTION HEAT TRANSFER IN HORIZONTAL CYLINDRICAL CAVITIES: A COMPUTATIONAL FLUID DYNAMICS (CFD) INVESTIGATION.

**Daniele Ludovisi**
Sargent & Lundy LLC
Nuclear Plant Analytical Group
55 E. Monroe St., Chicago, Illinois 60603, USA
Email: daniele.ludovisi@sargentlundy.com

**Ivo A. Garza**
Sargent & Lundy LLC
Nuclear Plant Analytical Group
55 E. Monroe St., Chicago, Illinois 60603, USA
Email: ivo.a.garza@sargentlundy.com

## ABSTRACT

*Many processes in power plants involve the storage and transfer of fluids including water in outdoor pipelines. Under extreme cold weather conditions, water can freeze if allowed to cool down to the freezing temperature. Installing insulation and maintaining adequate flow rate can sometimes prevent problems. However, during extended non-processing times, there are circumstances where cool down cannot be avoided and heat tracing along the piping becomes a necessity.*

*In many instances, the need for the installation of heat tracing is simply determined based on pipe size. However, by performing accurate calculations, it is possible to determine if the need for heat tracing is real or not, thus saving on installation and maintenance costs. Correlations for the estimation of the heat transfer coefficient in horizontal cavities are not sufficiently documented in literature. In the present work, two-dimensional CFD models are used to investigate the natural convection in water-filled horizontal pipes of different diameters. The analysis has been carried out based on the assumption of a uniform pipe surface temperature. The Nusselt number is estimated as a function of the Rayleigh number and shown not to be strongly dependent on the Prandtl number.*

*The analysis and the results of the numerical investigation are presented and compared to experimental data and other correlations available in literature. The documented correlation has an expanded range of applicability to high and low Rayleigh numbers, is supported by numerical and experimental results and is expressed in a simple form.*

## NOMENCLATURE

| | |
|---|---|
| $c_p$ | Specific heat of the fluid; |
| $D$ | Pipe inside diameter; |
| $g$ | Gravitational acceleration; |
| $h$ | Convection heat transfer coefficient; |
| $J_0$ | $0^{th}$ Bessel function of the first kind; |
| $J_1$ | $1^{st}$ Bessel function of the first kind; |
| $k$ | Thermal conductivity of the fluid; |
| $L$ | Pipe length; |
| $m$ | Integer counter: 1, 2, 3, ...; |
| $Nu$ | Nusselt number; |
| $Pr$ | Prandtl number; |
| $q'$ | Heat transfer per unit length of the pipe; |
| $r$ | Pipe radial coordinate; |
| $R$ | Pipe inside radius; |
| $Ra$ | Rayleigh number; |
| $t$ | Time; |
| $T_{average}$ | Fluid average temperature; |
| $T_s$ | Pipe surface temperature; |
| $\alpha$ | Thermal diffusivity of the fluid; |
| $\beta$ | Thermal expansion coefficient of the fluid; |
| $\gamma_m$ | $m^{th}$ positive roots of the equation $J_0(\gamma R) = 0$; |
| $\mu$ | Dynamic viscosity of the fluid; |
| $\nu$ | Kinematic viscosity of the fluid; |
| $\rho$ | Density of the fluid. |

## INTRODUCTION

Numerous correlations have been proposed for the overall heat transfer by natural convection on the outer surface of horizontal cylinders and in the space between concentric cylinders. However, despite their application in a variety of engineering applications, natural convection within horizontal cylindrical cavities is not sufficiently documented. Many industrial processes involve the storage and transfer of fluids including water in outdoor pipelines. During extended non-processing times in extreme cold weather conditions, still fluid in pipes can cool down and freeze.

This study combines the results of conduction and convection heat transfer analyses and experimental data to develop a correlation for the prediction of heat transfer from a fluid inside a horizontal cylinder under quasi-steady conditions. One of the applications of the results of this study is in the economic evaluation of heat tracing of horizontal pipes.

## ANALYSIS

Governing non-dimensional parameters of natural convection in closed cavities are the Prandtl ($Pr$) and Rayleigh ($Ra$) numbers [1]. $Pr$ is the fundamental characteristic for the thermal boundary layer for natural and forced convection. $Ra$ measures the relative effect of the buoyancy force, which drives the natural convection, and the viscous force, which tends to slow down the flow. It incorporates the influence of the fluid properties, cavity size and the driving force, buoyancy. The greater $Ra$, the greater the buoyancy effect and the more turbulent the flow. For small $Ra$, the flow is laminar and, for $Ra$ approaching zero, the fluid becomes still thus behaving similarly to a solid. The dimensionless form of the heat transfer coefficient is the Nusselt number $Nu$, which is defined as the ratio of convection heat transfer to fluid conduction heat transfer under the same conditions. In the present work, the Prandtl, Rayleigh and Nusselt numbers are defined as follows:

$$Pr = \frac{\nu}{\alpha} \tag{1}$$

$$Ra = \frac{g\beta(T_{average} - T_s)D^3}{\nu\alpha} \tag{2}$$

$$Nu = \frac{q'}{\pi k(T_{average} - T_s)} = \frac{hD}{k} \tag{3}$$

with

$$q' = \pi D h(T_{average} - T_s) \tag{4}$$

where $T_{average}$ is the average temperature of the fluid in the pipe (cross-sectional area), $T_s$ is the temperature of the pipe wall (uniform), and $q'$ is the heat transfer per unit length of the pipe.

This study investigates the Nusselt number as a function of the Rayleigh and Prandtl numbers:

$$Nu = f(Pr, Ra) \tag{5}$$

CFD modeling is used to simulate the fluid flow and heat transfer within two-dimensional horizontal cylindrical cavities ($L/D > 2.5$). Five different pipe diameters are analyzed: 1 inch, 2 inches, 4 inches, 8 inches and 16 inches. With the Rayleigh number being a function of the pipe diameter to the third power, this set of pipe diameters allows the evaluation of the system thermo-fluid dynamics for $Ra$ ranging from $10^3$ to $10^{10}$. In all simulations, the Boussinesq approximation is used. For $Ra > 10^6$, the flow is turbulent in which case the transient $SST\ k-\omega$ turbulent model is employed. The commercial code STAR-CCM+ Version 6.04 [2] is used for the simulations. For $Ra < 3\cdot 10^4$, the Nusselt number shows an asymptotic behavior due to the reduced velocity in the cavity. In the limit $Ra \to 0$, the heat transfer is analyzed by using equations for the heat conduction in a solid media.

## RESULTS AND DISCUSSION

The numerical simulations documented in this study consider water as process fluid with the properties as shown in Table 1, which are held constant throughout each simulation. However, the results are provided in dimensionless form and thus they can be generalized for other Newtonian fluids, temperatures and pipe sizes.

| $Pr$ $[-]$ | $\rho$ $[\frac{kg}{m^3}]$ | $c_p$ $[\frac{J}{kg\,K}]$ | $k$ $[\frac{W}{m\,K}]$ | $\mu$ $[\frac{N\,s}{m^2}]$ | $\beta$ $[\frac{1}{K}]$ |
|---|---|---|---|---|---|
| 1 | 859.4 | 4479.9 | 0.623 | $1.38E-4$ | $3.91E-04$ |
| 6 | 997.6 | 4186.4 | 0.620 | $8.89E-4$ | $3.91E-04$ |
| 15 | 999.9 | 4224.5 | 0.561 | $1.99E-3$ | $3.91E-04$ |

**TABLE 1**. Fluid properties used in the CFD simulations.

The system is initialized with a uniform high fluid temperature and zero angular and radial velocity. A uniform low pipe surface temperature is applied as boundary condition (cooling process). At each time step of the transient analysis, the overall heat loss from the fluid and the average fluid temperature are computed. The Rayleigh and Nusselt numbers are computed based on Equations 1 to 3. Three different Prandtl numbers are evaluated which are typical for liquid water: 1, 6 and 15 (see Table 1).

A review of the numerical results shows the presence of four periods in the simulated transient cooling process: an initial regime of heat conduction (Period 1), a developing flow regime (Period 2), a quasi steady-state regime (Period 3) and, finally, an asymptotic quasi heat conduction regime (Period 4) (see Figure 1). The developed quasi steady-state natural convection process is characterized by the presence of a boundary layer near the pipe wall and a central core with almost isothermal horizontal sections and a stratified temperature profile in the vertical direction (see Figures 2 and 3). As seen in Figure 1, the calculated Nusselt number as function of Rayleigh number rapidly decreases (Period 1), stabilizes by slightly increasing (Period

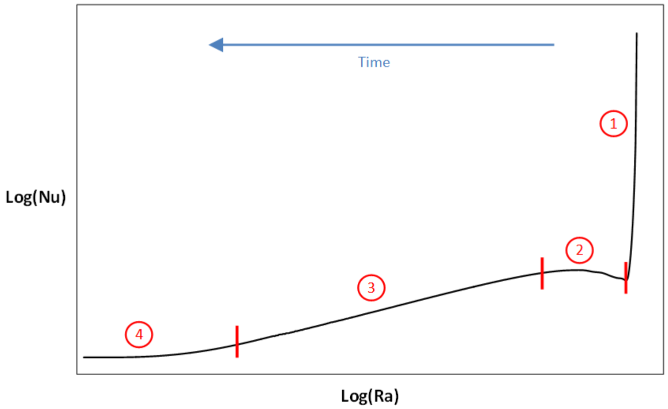


**FIGURE 1**. Typical $Nu$ vs. $Ra$ trend in the transient simulations

2), then slowly and steadily decreases (Period 3), and finally approaches an asymptotic value (Period 4). The duration of each period is different and depends on the mass of water in the pipe. The quasi steady-state regime (Period 3) is the focus of the numerical analysis presented in this study. The asymptotic values for Period 4 are theoretically investigated based on conduction heat transfer principles.

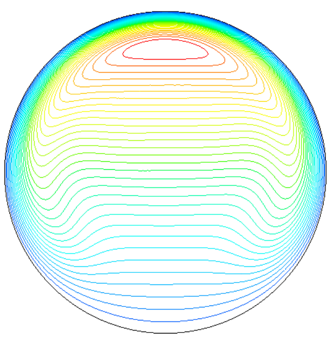

**FIGURE 2**. Typical isotherms at low Rayleigh numbers

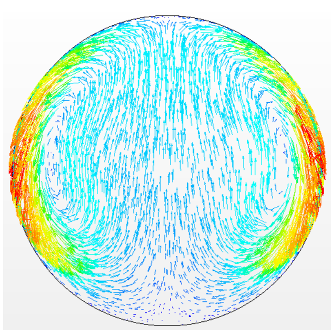

**FIGURE 3**. Typical velocity distribution at low Rayleigh numbers

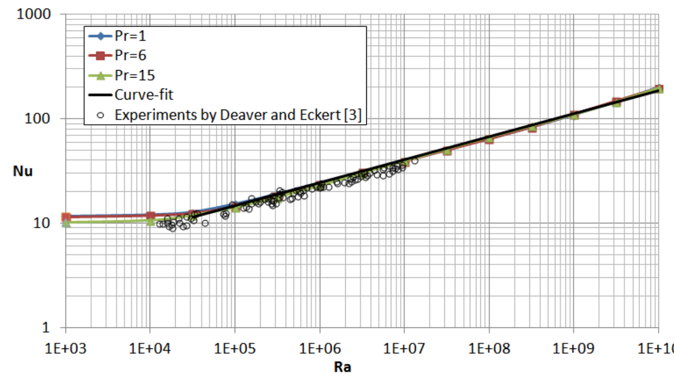


**FIGURE 4**. Nusselt number as a function Rayleigh number for various Prandtl numbers.

Figure 4 shows the results of the numerical CFD simulations (Period 3). As seen, the Nusselt number is a weak function of the Prandtl number for $1.0 \leq Pr \leq 15.0$. Therefore, the effect of the Prandtl number is ignored. The results of the numerical analyses are curve-fit and, for $3 \cdot 10^4 \leq Ra \leq 1 \cdot 10^{10}$, the Nusselt and Rayleigh numbers are correlated by the following equation:

$$Nu = 1.15 \cdot Ra^{0.22} \tag{6}$$

Figure 4 also shows experimental results for various mixtures of water and glycerol by Deaver and Eckert [3]. As seen, there is a good agreement between the numerical and experimental results. The error band of the proposed correlation is $\pm 10\%$ on the numerical results and $\pm 20\%$ on the experimental data [3]. The experimental results also confirm that the Prandtl number does not have a significant impact on the Nusselt number. Kuehn and Goldstein [4] also proposed a Nusselt number correlation as a function of the Rayleigh number. This correlation was based on engineering judgment by combining various expressions available in literature for configurations somewhat comparable to a horizontal cavity (natural convection within a sphere, concentric cylinders, etc.). The documented equation was supported by the experimental data only in a narrow range of Rayleigh numbers (Deaver and Eckert [3]). The error band of the correlation proposed in the present study is $\pm 20\%$ on the correlation derived by Kuehn and Goldstein [4] in the range of $Ra$ supported by experimental evidence. Martynenko and Khramtsov [5] reported in their work a correlation which is based on the expression originally published by Kuehn and Goldstein [4] and added a correction factor which is function of the Prandtl number. However, the Nusselt numbers calculated by employing the correlation by Martynenko and Khramtsov [5] do not agree with the experimental results of Deaver and Eckert [3], the results by Kuehn and Goldstein [4] and the results obtained in the present paper.

As the Rayleigh number approaches small values, the Nusselt number approaches an asymptotic value (see Figures 1 and

4). With the water velocity being small, conduction heat transfer equations are used to estimate this asymptotic limit. The theoretical temperature distribution in a solid cylinder as function of time and position is well known in a closed form [6]:

$$T(r,t) = T_s + \frac{2}{R^2}\sum_{m=1}^{\infty} e^{-\alpha\gamma_m^2 t}\cdot\frac{J_0(\gamma_m r)}{J_1^2(\gamma_m R)}\cdot\int_{r'=0}^{R} r' J_0(\gamma_m r')F(r')dr' \quad (7)$$

where $J_0$ and $J_1$ are Bessel functions of the first kind, $\gamma_m$ are positive roots of the transcendental equation $J_0(\gamma R) = 0$ and $F(r) = T_0(r) - T_s$ is the initial condition. In this study, the initial temperature distribution $T_0(r)$ is set to be equal to a constant $T_0$ for simplification of the calculations and it does not affect the results of the analysis.

The Nusselt number is calculated using Equation 3 as $Nu = q'/\pi k(T_{average} - T_s)$. $T_{average}$ and $q'$ are calculated by manipulation of Equation 7, and the expression for $T_{average}$ is derived as follows:

$$T_{average}(t) = T_s + \frac{\int_{r'=0}^{R} 2\pi r'[T(r',t) - T_s]dr}{\pi R^2} \quad (8)$$

or:

$$T_{average}(t) = \sum_{m=1}^{\infty}\frac{4(T_0 - T_s)}{R^2\gamma_m^2}e^{-\alpha\gamma_m^2 t} \quad (9)$$

The expression for $q'$ is derived as follows:

$$q'(t) = -2\pi Rk\left.\frac{\partial T(r,t)}{\partial t}\right|_{r=R} \quad (10)$$

or:

$$q'(t) = \sum_{m=1}^{\infty} 4\pi k(T_0 - T_s)e^{-\alpha\gamma_m^2 t} \quad (11)$$

With the above definitions, the following expression for the Nusselt is obtained:

$$Nu(t) = \frac{R^2\sum_{m=1}^{\infty} e^{-\alpha\gamma_m^2 t}}{\sum_{m=1}^{\infty}\frac{e^{-\alpha\gamma_m^2 t}}{\gamma_m^2}} \quad (12)$$

As seen in the above equation, the Nusselt number is a function of time. As time tends to infinity, the temperature distribution in the solid media is expected to be similar to the distribution in the fluid at the end of Period 4, with a warmer core region and a cold outer boundary. Taking the limit of the Nusselt number function in Equation 12 for $t \to \infty$, the asymptotic value for Period 4 is found. Numerical calculations show that this asymptotic value is not dependent on $Pr$ or pipe size and is equal to:

$$\lim_{t\to\infty} Nu(t) = 5.78 \quad (13)$$

## CONCLUSIONS

A correlation for the prediction of natural convection heat transfer from a fluid inside a horizontal cylinder with a uniform wall temperature is reported in this study. The Nusselt number is found to be a function of the Rayleigh number according to the following equation:

$$Nu = 1.15\cdot Ra^{0.22} \quad (14)$$

which is valid for $3\cdot 10^4 \le Ra \le 10^{10}$ and $1 \le Pr \le 15$. Additionally, the Nusselt number is found to have an asymptotic value of 5.78 for $Ra \to 0$.

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